$\| \varphi - \varphi^* \|_{C_\alpha} < \varepsilon$, where ε is a sufficiently small positive number, then it is said that the direction λ^* is a normal perturbation of the direction λ.

In addition to the problem (2.1), (1.1), let us consider the normally perturbed problem of the form

$$\frac{\partial}{\partial \bar{\zeta}} W - A^*(\zeta) W - \overline{B^*(\zeta)}\, \overline{W} = F^*(\zeta), \qquad \zeta \in G$$

$$\mathrm{Re}\left[W \frac{d\tau}{ds} \frac{d\tau}{d\lambda^*} \right] = h^*$$

$$\| A - A^* \|_{L_p} < \varepsilon, \quad \| B - B^* \|_{L_p} < \varepsilon, \quad \| F - F^* \|_{L_p} < \varepsilon$$

$$\| \varphi - \varphi^* \|_{C_\alpha} < \varepsilon, \quad \| h - h^* \|_{C_\alpha} < \varepsilon$$

It is clear that if the boundary problem (1.1), (2.1) is quasi-correct, then the normally perturbed problem is also quasi-correct for sufficiently small ε.

In conclusion, let us examine the case when the vector λ belongs to the class s. Then $\chi = 2(m - 1)$. Therefore, $\chi = -2$, $n = 0$, $n' = 3$ for simply connected shells. This means that three conditions of the form (3.1) should be satisfied in order to realize a membrane state in a shell with one hole. If the contour L of the middle surface passes along an isometrically conjugate line, then the corresponding surface bendings will be trivial [2] and the formulated problem has a unique solution. For shells with three or more holes the membrane state will be quasi-correct since $n = 0$ and $n' = 3m - 3$. For doubly connected shells of positive curvature the membrane state cannot always be realized; however, if the hole contours of the shell coincide with isometrically conjugate lines on the middle surface, then such a state is realized unconditionally.

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ADDENDUM TO THE PAPER "ON CERTAIN EXACT SOLUTIONS OF THE FOURIER EQUATION FOR REGIONS VARYING WITH TIME"

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The exact solutions given in [1] are generalized to the case of cylindrical and spherical sectors rotating about the azimuth relative to the coordinate origin either at a uniform rate or with uniform acceleration (or deceleration). The

class of equations of motion of the boundaries of the half-space (in the Cartesian coordinates) which lead to exact solutions of the Fourier equation defined in these domains, is enlarged.

1. Cartesian coordinates. Let the domain in which the Fourier equation is defined be semi-bounded in the coordinate x_i ($i = 1$ or $i = 1,2$ or $i = 1, 2, 3$), i.e. $x_i \in (R_i^0(t), \infty)$ and let the function η_i appearing in the formula (1.3) of [1] be an auxilliary function with continuous first and second order derivatives. Then the formulas (1.4) – (1.6) of [1] determine the form of the function η_i relative to the law of motion of the boundary $R_i^{(0)}$, for which the heat conduction equation written in the Cartesian coordinates admits the exact solution obtained by separation of variables.

2. Cylindrical coordinates. Exact solutions given in Sect. 2 of [1] admit a generalization to cylindrical sectors rotating at a uniform rate or with a uniform acceleration (deceleration). Motion in the z-direction can either be absent or belong to one of the types of motion determined by the formulas (1.4) – (1.6) of [1].

In fact, let us introduce in the equation

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial U}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 U}{\partial \varphi^2}+\frac{\partial^2 U}{\partial z^2}=\frac{\partial U}{\partial t}-f(r,\varphi,z,t)$$

$$r\in(R_1(t),\alpha R_1(t)),\qquad \varphi\in(R_2^{(0)}(t),R_2^{(1)}(t)),\qquad z\in(R_3^{(0)}(t),R_3^{(1)}(t))$$

where $\alpha = \text{const}$ (in particular, zero or ∞) and $R_2^{(1)} - R_2^{(0)} = \text{const}$, the following new coordinates

$$y_1 = r / R_1,\quad y_2 = \varphi - R_2^{(0)},\quad y_3 = (z - R_3^{(0)}) /$$

where $\eta = R_3^{(1)} - R_3^{(0)}$ if $|R_3^{(1)}|_{t<\infty} < \infty$ or η is some auxilliary function if $R_3^{(1)} \equiv \infty$. We also introduce a new function V

$$U = qV,\qquad q = (R_1\sqrt{\eta})^{-1}\exp\Big[-\frac{1}{4}(y_1^2R_1R_1^{\cdot} +$$

$$2y_2R_2^{(0)\cdot} + y_3\eta\eta^{\cdot} + 2y_3\eta R_3^{(0)\cdot} + \int^t [R_2^{(0)\cdot 2} + R_3^{(0)\cdot 2}]\,dt$$

(where q is determined using the method given in [2]). Then we arrive at the following equation in V:

$$\frac{1}{R_1^2}\left[\frac{1}{y_1}\frac{\partial}{\partial y_1}\left(y_1\frac{\partial V}{\partial y_1}\right)+\frac{1}{4}y_1^2R_1^3R_1^{\cdot\cdot}V+\frac{1}{y_1^2}\left(\frac{\partial^2 V}{\partial y_2^2}+\frac{1}{2}y_2R_2^{(0)\cdot\cdot}V\right)\right]-$$

$$\frac{1}{\eta^2}\left[\frac{\partial^2 V}{\partial y_3^2}+\frac{1}{4}\left(y_3^2\eta^3\eta^{\cdot\cdot}+2y_3\eta^3R_3^{(0)\cdot\cdot}\right)V\right]=$$

$$\frac{\partial V}{\partial t}-\frac{1}{q}f\left(y_1R_1,\ y_2+R_2^{(0)},\ y_3\eta+R_3^{(0)}\right)$$

The above equation admits an exact solution obtained by separating the variables, if the conditions

$$R_1^3R_1^{\cdot\cdot} = \text{const},\quad R_2^{(0)\cdot\cdot} = \text{const},\quad \eta^3\eta^{\cdot\cdot} = \text{const},\quad \eta^3R_3^{(0)\cdot\cdot} = \text{const}$$

hold simultaneously. The assertion made above now follows from the second condition.

3. Spherical coordinates. 3.1. Let the heat conduction equation depend only on a single spatial coordinate r. As we know, in this case we can introduce a new function $W = rU$ (U is the unknown function) and obtain an equation in W which is analogous to the Fourier equation in the Cartesian coordinates. Consequently the results obtained in Sect. 1 are applicable in this case.

3.2. Let the domain be a spherical sector

$$r \in (R_1(t), \alpha R_1(t)), \quad \varphi \in (R_2(t), R_2(t) + \beta), \quad \vartheta \in (\gamma, \delta)$$

where α, β, γ and δ are constants.

Performing the manipulations analogous to those in Sect. 2 we can confirm that the initial equation admits, as before, an exact solution, provided that the sector varies its radial dimensions according to the equation $R_1^3 R_1^{\cdot\cdot} = \text{const}$, at the same time rotating by the angle φ about the coordinate origin at a uniform rate or with a uniform acceleration (deceleration), i.e. when $R_2(t) = Mt^2 + At + B$ (M, A and B are constants).

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